PCF $\supseteq$ λ-cálculo

términos t, u, v, w

Variables x, y, z

$t ::=$
- $x, y, z \in Var$
- $\lambda x.t$
- $t\,u$

λ-cálculo

Anonimas

$\lambda x.x$

id (x) { x }

fst(x, y) { x }

fst (id, ?) { x }

f(?) { id }

Un sólo parámetro

$\lambda x. \lambda y. x$

$(\lambda x. \lambda y. x)(\lambda z.z)$?

$\lambda y. \lambda z.z$

Enteros de Church

$n = \lambda f. \lambda x. \underbrace{f \dots f}_{n}\, x$

$\underline{0} = \lambda f. \lambda x. x$

$\underline{1} = \lambda f. \lambda x. f\,x$

$\underline{2} = \lambda f. \lambda x. f\,f\,x$

BNF LFyA

PCF

$t ::= x \mid \lambda x.t \mid t\,t \mid n \in \mathbb{N} \mid t+t \mid t-t \mid t \times t \mid t/t \mid \text{ifz } t \text{ then } t \text{ else } t$
$\mid \mu x.t \mid \text{let } x = t \text{ in } t$

$\boxed{(\lambda x. x \times x)\,2} = 2 \times 2 = \boxed{4}$

$$\frac{\text{let } x = u \text{ in } t}{(\lambda x.t)\,u}$$

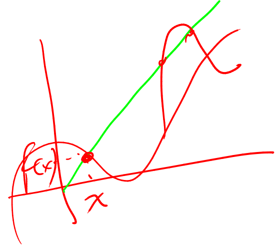
$\text{Fact} = \mu f. \underbrace{\lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times f\,(x-1)}_{G}$

$0! = 1$

$n = n \times (n-1)!$

$(t\,u)\,r$

$$\underbrace{\mu f. G}_{Fact} = (\lambda f. G)(\mu f. G)$$



$f(x_0) = x_0$

$$
\begin{aligned}
\underline{Fact\ 2} &= (\mu f. G)\ 2 \\
&= ((\lambda f. G)(\mu f. G))\ 2 \\
&= ((\lambda f. \lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times f(x-1))\ \underline{Fact})\ 2 \\
&= ((\lambda f. \lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times Fact\ (x-1))\ )\ 2 \\
&= (\lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times Fact\ (x-1))\ 2 \\
&= \text{ifz } 2 \text{ then } 1 \text{ else } 2 \times Fact\ (2-1) \\
&= 2 \times Fact\ (2-1) = 2 \times Fact\ 1 \\
&= 2 \times \underbrace{Fact}_{\mu f. G}\ (x-1))\ 1 \\
&= 2 \times (\lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times Fact\ (x-1))\ 1 \\
&= 2 \times \text{ifz } 1 \text{ else } 1 \times Fact\ (1-1) \\
&= 2 \times 1 \times Fact\ (1-1) \\
&= 2 \times 1 \times \underline{Fact}\ 0 \\
&= 2 \times 1 \times (\lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times \underline{Fact\ (x-1)})\ 0 \\
&= 2 \times 1 \times \text{ifz } 0 \text{ then } 1 \text{ else} \ldots \\
&= 2 \times 1 \times 1 = 2
\end{aligned}
$$

$$(\mu f. G) = (\lambda f. G)\ \underline{Fact}$$

$$2 + 1 \rightarrow 3$$

$$1 + 1 + 1 = 3$$

Semántica operacional

$\rightarrow$ Reescritura

Reducción

$$(\lambda x. x \times x)\ (\mu f. \lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times f(x-1))\ 5$$

$$(Fact)$$

$$(Fact\ 5) \times (Fact\ 5)$$

$\boxed{(\lambda x.t)u} \to \boxed{t[u/x]}$ $\quad \otimes = +,-,\times,/$

$n \otimes m \to p \quad$ si $\quad n \otimes m = p$ $\qquad (\lambda x.\, x \times x)\, 2 \to 2 \times 2$

$\text{ifz } 0 \text{ then } t \text{ else } u \to t$

$n > 0 \quad \text{ifz } n \text{ then } t \text{ else } u \to u$ $\qquad (\lambda x.t)\, u$

$\mu f.t \to t[\mu f.t/f]$ $\qquad \mu f.t = (\lambda f.t)\, \mu f.t$

$\text{let } x = u \text{ in } t \to t[u/x]$ $\qquad t[\mu f.t/f]$

$((\lambda x.\lambda y.\, x+y)\, 2)\, 3 \to (\lambda y.\, 2+y)\, 3$

$2+3 \to 5$

$t\,r\,s = (t\,r)\,s$

$(\lambda x.t)\,u$

$\lambda x.(t\,u) = \lambda x.\,t\,u$

$$\frac{t \to u}{t\,s \to u\,s} \qquad \frac{t \to u}{s\,t \to s\,u}$$

$t ::= x \mid \lambda x.t \mid t\,t \mid n \mid t \otimes t \mid \text{ifz } t \text{ then } t \text{ else } t \mid \mu x.t \mid \text{let } x = t \text{ in } t$

$$\frac{t \to u}{\lambda x\, t \to \lambda x\, u}$$

$f(x = p)\ \{ \ldots \}$

$$\frac{t \to u}{t \otimes s \to u \otimes s} \qquad \frac{t \to u}{s \otimes t \to s \otimes u}$$

$((\lambda x.x)\, 3) + 1 \to \boxed{\boxed{3} + 1} \to 4$

Ejercicio: escribir las que faltan

$\boxed{(\lambda x.t)\,u \to t[u/x]}$ $\qquad \boxed{\mu x.t \to t[\mu x.t/x]}$

$$\underbrace{\mu x. x+x}_{t} \to \underbrace{\underbrace{(\mu x. x+x)}_{x} + \underbrace{(\mu x. x+x)}_{x}}_{t} \to (\mu x. x+x) + (\mu x. x+x) + (\mu x. x+x) + (\mu x. x+x)$$

$$\Omega = \boxed{\mu x. x} \to \mu x. x \to \mu x. x \to \mu x. x \to \dots$$

$$\Omega = \underbrace{(\lambda x. xx)}_{(\lambda x. t)} \underbrace{(\lambda x. xx)}_{v} \to \underbrace{\underbrace{(\lambda x. xx)}_{x} \underbrace{(\lambda x. xx)}_{x}}_{t[v/x]} \to$$

$$(\mu f. \lambda x. f x)\, 0 \to (\lambda x. (\mu f. \lambda x. f x)\, x)\, 0$$

$$\to (\mu f. \lambda x. f x)\, 0$$

λ	fun
μ	fix

$$\underbrace{(\mu f. t)}_{x_0} \to t[\mu f. t/f] \leftarrow \underbrace{(\lambda f. t)}_{F} \underbrace{(\mu f. t)}_{(x_0)}$$

$$Y = \lambda f. (\lambda x. f(xx))(\lambda x. f(xx))$$

$$x_0 \to F(x_0)$$

$$Y F \to (\lambda x. F(xx))(\lambda x. F(xx))$$

$$\to \underbrace{F}\ \underbrace{((\lambda x. F(xx))(\lambda x. F(xx)))}_{x_0}$$

$$\boxed{\underbrace{\mu f. G}_{YF} = \underbrace{(\lambda f. G)}_{F} \underbrace{(\mu f. G)}_{YF}}$$

$$\boxed{\mathit{Fact} = \mu f. \underbrace{\lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times f(x-1)}_{G}}$$

$$Y(\lambda f. \lambda x. \text{ifz } x \text{ then } 1 \text{ else } x \times f(x-1))$$

$$Y = \lambda f.(\lambda x.f(xx))\ \lambda x.f(xx))$$

$$Y(\lambda f.\lambda x.\text{if } z\ x \text{ then } 1 \text{ else } x * f(x-1))$$

$\Delta\Delta$ = Fact

$$Y(\lambda f.G) \longrightarrow (\lambda x.(\lambda f.G)(xx))\ (\lambda x.(\lambda f.G)(xx))$$

$$\longrightarrow (\lambda f.G)(\Delta\Delta) \longrightarrow \lambda x.\text{if } z\ x \text{ then } 1 \text{ else } x * \Delta\Delta(x-1)$$

$$(\lambda x.\lambda x.x)\ 2\ 3 \longrightarrow (\lambda x.\cancel{x})\ 3 \longrightarrow 3$$

$$(\lambda x.\lambda x.x)\ 2\ 3 \longrightarrow (\lambda x.x)\ 3$$

$f(x=2)\{\text{id}\}$

$(\lambda x.x)\ [2/x]$

$\boxed{t[v/x]}$

$$(\lambda x.\lambda y.(\lambda x.x+y)\ x)\ 5\ 4$$

$$\longrightarrow (\lambda y.(\lambda x.x+y)\ 5)\ 4 \longrightarrow (\lambda x.x+4)\ 5$$

$$\longrightarrow (\lambda y.5+y)\ 4 \longrightarrow 5+4 \rightarrow 9$$

$\boxed{\text{let } x=v \text{ in } t \rightarrow t[v/x]}$

---

let $x=4$ in
(let $f=\lambda y.y+x$ in
let $x=5$ in
$f\,6$) $\longrightarrow$ let $f=\lambda y.y+4$ in
let $x=5$ in
$f\,6$ $\rightarrow$ let $x=5$ in
$(\lambda y.y+4)\ 6$
$\downarrow$
$(\lambda y.y+4)\ 6 \rightarrow 6+4 \rightarrow 10$

let $x=4$ in
let $f=\lambda y.y+x$ in $f\,6$

let $x=4$ in $(\lambda y.y+x)\ 6 \longrightarrow$ let $x=4$ in $6+x \rightarrow 6+4 \rightarrow 10$

let $x=4$ in
let $x=5$ in
$(\lambda y.y+x)\ 6$

Lisp

11

$t[v/x]$

$$x[u/x] = u$$
$$y[u/x] = y$$
$$(\lambda x.t)[u/x] = \lambda x.t$$
$$(\lambda y.t)[u/x] = \lambda y.\, t[u/x] \quad \text{si } y \notin FV(u)$$
$$(\lambda y.t)[u/x] = \lambda z.\, t[z/y][u/x] \quad \text{si } y \in FV(u),\ z \text{ fresh}$$
$$(t\, r)[u/x] = t[u/x]\ r[u/x]$$
$$n[u/x] = n$$
$$(t \otimes r)[u/x] = t[u/x] \otimes r[u/x]$$
$$(\text{ifz } t_1 \text{ then } t_2 \text{ else } t_3)[u/x] = \text{ifz } t_1[u/x] \text{ then } t_2[u/x] \text{ else } t_3[u/x]$$

$$(\mu x.t)[u/x] = \mu x.t$$
$$(\mu y.t)[u/x] = \mu y.\, t[u/x] \quad \text{si } y \notin FV(u)$$
$$(\mu y.t)[u/x] = \mu z.\, t[z/y][u/x] \quad \text{si } y \in FV(u),\ z \text{ fresh}$$

$$(\text{let } x = r \text{ in } t)[u/x] = \text{let } x = r[u/x] \text{ in } t$$
$$(\text{let } y = r \text{ in } t)[u/x] = \text{let } y = r[u/x] \text{ in } t[u/x] \quad \text{si } y \notin FV(u)$$
$$(\text{let } y = r \text{ in } t)[u/x] = \text{let } z = r[u/x] \text{ in } t[z/y][u/x] \quad \text{si } y \in FV(u),\ z \text{ fresh}$$

$$((\lambda y.t)\, r)[u/x]$$
$$((\lambda z.\, t[z/y])\, r)[u/x]$$

$$\lambda y.\ (\lambda y.t)[y/x]$$
$$\lambda y.\, t[y/x]$$
$$(\lambda x.\lambda x.t)\, 3 \to (\lambda x.t)[3/x] = \lambda x.t$$

$$\lambda y.\ ((\lambda x.\lambda y.\, y+x)(y+1))$$
$$\to \boxed{\lambda y}.\ (\lambda y.\, y+x)[y+1/x]$$
$$\lambda y.\ (\lambda y.\, y+y+1)$$
$$(\lambda y.\, y+x)[y+1/x]$$
$$= (\lambda z.\, z+x)[y+1/x]$$
$$\lambda z.\, z+y+1$$

$$(\text{let } x = r \text{ in } t)[u/x]$$
$$\lambda x.$$
$$((\lambda x.t)\, r)[u/x]$$
$$(\lambda x.t)\ r[u/x]$$
$$((\lambda x.t)\, r)[u/x]$$
$$(\lambda x.t)\cancel{[u/x]}\ r[u/x]$$

$$\text{let } x = 4 \text{ in } (\text{let } f = \lambda y.y+x \text{ in let } x = 5 \text{ in } f\, 6) \longrightarrow \text{let } x = 4 \text{ in } (\text{let } x = 5 \text{ in } f\,6)[\lambda y.y+x/f]$$

$$\text{let } x = 4 \text{ in } (\text{let } z = 5 \text{ in } f\,6)[\lambda y.y+x/f]$$

$FV(t)$ 1

$FV(x) = \{x\}$

$FV(\lambda x.t) = FV(t) - \{x\}$

$FV(tu) = FV(t) \cup FV(u)$

$FV(n) = \emptyset$

$FV(t \otimes u) = FV(t) \cup FV(u)$

$FV(\text{ifz } t \text{ then } u \text{ else } r) = FV(t) \cup FV(u) \cup FV(r)$

$FV(\mu x.t) = FV(t) - \{x\}$

$FV(\text{let } x = u \text{ in } t) = FV((\lambda x.t)\, u) = (FV(t) - \{x\}) \cup FV(u)$

$FV((\lambda x.x)\ x) = \{x\}$

$FV(\text{let } x = \lambda y.x+y \text{ in } x$

$FV((\lambda x.x)(\lambda y.\cancel{x}+y)) = \{x\}$