$t \rightsquigarrow r$

$\to^*$ $\quad \to$

$t \to^* r$

$t = v_0 \to v_1 \to v_2 \to \dots \to v_n = r$

$t \to^* t$

Def

$\to^*$ cierre reflexivo y transitivo de $\to$

$\to^+$ cierre transitivo de $\to$

$\to^*$

$(\lambda x.x+2)\,1 \to^* 3$

$(\lambda x.x+2)\,1 \to 1+2 \to 3$

$(\lambda x.x+2)\,1 \to^+ 3$

---

1. t está en "forma normal" si no existe u / $t \to u$
2. t es normalizable si existe u en f.n. tal que $t \to^* u$
3. t es fuertemente normalizable si no existe una secuencia infinita $v_0, v_1, \dots$ tal que $t \to v_0 \to v_1 \to \dots$

SN

3 $\quad \lambda x.x+1$

$(\lambda x.1)\,\Omega \to^* 1$

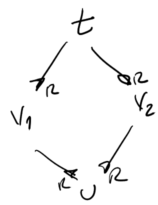
Ω no es normalizable

$(\lambda x.1)\,\Omega$ no es fuertemente normalizable

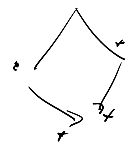
$(\lambda x.x+1)\,3$

$\to_R$ $\quad \to_R^*$

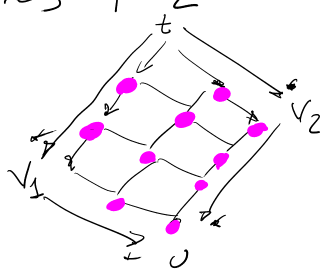
- $\to_R$ satisface la propiedad del diamante si $t \to_R v_1$ y $t \to_R v_2$ entonces $\exists u$ / $v_1 \to_R u$ y $v_2 \to_R u$



- $\to_R$ es Church-Rosser o confluente si $\to_R^*$ satisface $\diamond$: $t \to_R^* v_1$, $t \to_R^* v_2$ entonces $\exists u$ / $v_1 \to_R^* u$, $v_2 \to_R^* u$



- $\to_R$ tiene formas normales únicas si $t \to_R^* v_1$ y $t \to_R^* v_2$ tales que v_1 y v_2 están en forma normal, entonces $v_1 = v_2$



Lema

1. Si $\to_R$ satisface $\diamond$ entonces es CR
2. Si $\to_R$ satisface CR entonces tiene f.n.u

Teorema PCF satisface CR

Ejercicio: mostrar que PCF No satisface ◇

RedEx

$(\lambda x.(\lambda y.x+y)2)3$

$(\lambda y.3+y)\,2 \qquad (\lambda x.x+2)3$

$3+2$

5

Fact $= \mu f.\ \lambda n.$ if z n then 1 else $n * f(n-1)$ $= \mu f.G$

Fact 0 $= (\mu f.G)\,0 \rightarrow (G[\text{Fact}/f])\,0 \xrightarrow{G} G[G[\text{Fact}/f]/f]\ 0$

$\rightarrow G[G[G[\text{fact}/f]/f]/f]\ 0$

$\rightarrow \dots$

$C_0 = \lambda x.0$

$b_1 = (\mu f.\lambda x.fx)\,0 \longrightarrow \underbrace{(\lambda x.(\mu f.\lambda x.fx)x)\,0}_{b_2} \rightarrow b_1$

$C_0 b_1 = (\lambda x.0)\,b_1 \longrightarrow 0$

$(\lambda x.0)\,b_2 \longrightarrow (\lambda x.0)\,b_1 \longrightarrow (\lambda x.0)\,b_2 \longrightarrow (\lambda x.0)\,b_1$

let rec f x = f x in

let g x = 0 in

g (f 0)

$(\lambda x.\underline{x+1})\,\underline{b_1} \rightarrow \underline{b_1}+1$

Red Ex

Radical

débil

$(\lambda x.x+(4+5))\,3 \longrightarrow 3+(4+5) \longrightarrow 3+9 \longrightarrow 12$

$(\lambda x.x+9)3$

fuerte

$\boxed{\lambda x.x+(4+5)}$

$\cancel{\dfrac{t \rightarrow u}{\lambda x.t \rightarrow \lambda x.u}}$ débil

Call-by-name

Redex de + a la izq primero

$(\lambda x.0)\, b_1$

$\overset{I_1}{(\lambda x.x)}\overset{I_2}{(\lambda x.x)}\overset{I_3}{(\lambda x.x)} \to I_2 I_3$

Teorema Si un término es normalizante entonces CBN termina ✓

$(\lambda x.\, x+x)\ (\text{Fact } 1000) \to (\text{Fact } 1000) + (\text{Fact } 1000)$

$p + p$

$\boxed{\text{Fact } 1000} \leftarrow p$

Call-by-value

No "toman" el argumento hasta q' no sea un valor.

$\cancel{x}$

$\lambda x.t$

$\lambda x.\, (\lambda y.y)\, x$

$\text{let } x = (I2) \text{ in } x+x$

CBN: $(I_2) + (I2) \to 2 + (I2) \to 2+2$

CBV: $\text{let } x = 2 \text{ in } x+x \to 2+2 \to 4$

if (t) then (r) else (s)

PCF Tipado.

$(\lambda x. x+1)((\lambda z.z)((\lambda w.\lambda y.y)\,3)) \to^* (\lambda y.y)+1$

$\mathbb{F}+1$ $\quad$ $\mathbb{F}$ $\quad$ $\mathbb{F} \Rightarrow \mathbb{F}$ $\quad$ $\mathbb{N} \Rightarrow$ Fn. $\quad$ $\mathbb{N}$

$(\lambda x.x)\ 1$ $\quad$ $\mathbb{N}$ $\quad$ $\cancel{\lambda x.x}$

Funcion: Dom $\to$ Cod.

$f: \text{Pares} \to \mathbb{N}$

$f(x) \mapsto \frac{x}{2}$

$f(3+(y+1))$

$f(\text{Fact } 100)$

$A = \{X \mid X \notin X\}$

$A \in A$? $\quad$ X

$A \notin A$ $\quad$ X

Tipos

- nat es un tipo
- Si A y B son tipos entonces $A \Rightarrow B$ es un tipo

$(nat \Rightarrow nat) \Rightarrow nat$

$\lambda f. f\,1$

x

n

$\lambda x.t$

$\lambda f. f1 : (nat \Rightarrow nat) \Rightarrow nat$ $\quad$ Curry

$\lambda f: nat \Rightarrow nat.\ f1 : (nat \Rightarrow nat) \Rightarrow nat$ $\quad$ Church

func (int x) {

}

$\lambda x.x : A \Rightarrow A$ $\to$ $\lambda x:nat.x : nat \Rightarrow nat$

$\to$ $\lambda x:(nat \Rightarrow nat).x : (nat \to nat) \Rightarrow (nat \Rightarrow nat)$

$A ::= nat \mid A \Rightarrow A$

$t ::= x \mid \lambda x:A.t \mid t\,t \mid n \mid t \otimes t \mid \text{ifz } t \text{ then } t \text{ else } t \mid \mu x:A.t \mid \text{let } x:A = t \text{ in } t$

$\boxed{\Gamma \vdash t : A}$

$y : nat \Rightarrow (nat \Rightarrow nat) \vdash \lambda x:nat.\, y\, x : nat \Rightarrow (nat \Rightarrow nat)$

$nat \Rightarrow ?$

$nat \Rightarrow (nat \Rightarrow nat) \vdash nat \Rightarrow (nat \Rightarrow nat)$

$\Gamma, x:A \overset{def}{=} \Gamma \cup \{x:A\}$

$x \mapsto A$

$\Gamma(x) = A$

$(\Gamma, x:A)(x) = A$

$$\frac{}{\Gamma, x:A \vdash x:A}\,ax_v$$

$$\frac{}{\Gamma \vdash n : nat}\,ax_c$$

$$\frac{\Gamma, x:A \vdash t : B}{\Gamma \vdash \lambda x:A.t : A \Rightarrow B}\Rightarrow_i$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B}$$

$$\frac{\Gamma \vdash t : A \Rightarrow B \qquad \Gamma \vdash r : A}{\Gamma \vdash t\,r : B}\Rightarrow_e$$

$$\frac{\Gamma \vdash A \Rightarrow B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

$$\frac{\dfrac{}{x:nat \vdash x:nat}\,ax}{\vdash \lambda x:nat.x : nat \Rightarrow nat}\Rightarrow_i$$

$$\frac{\Gamma \vdash t : nat \qquad \Gamma \vdash r : nat}{\Gamma \vdash t \otimes r : nat}\otimes$$

$\lambda x.t$

n

$I\,n$

$$\frac{\Gamma \vdash t : nat \qquad \Gamma \vdash r : A \qquad \Gamma \vdash s : A}{\Gamma \vdash \text{ifz } t \text{ then } r \text{ else } s : A}\,Ifz$$

$$\frac{\Gamma, x:A \vdash t : B \qquad \Gamma \vdash r : A}{\Gamma \vdash \text{let } x:A = r \text{ in } t : B}\,let$$

$(\lambda x:A.t)\, r$

$$\frac{\dfrac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

$$\frac{\Gamma, A \vdash B \qquad \Gamma \vdash A}{\Gamma \vdash B}$$

$$\frac{\Gamma, x:A \vdash t : B}{\Gamma \vdash \mu x:A.t : (A\Rightarrow B)\Rightarrow B}\ fix$$

$$(\lambda x:A.t)(\mu x:A.t) \rightarrow t[\mu x:A.t/x]$$

$A \Rightarrow B$ $\quad$ $A\Rightarrow B$ $\quad$ A

No

$$\frac{\Gamma, x:A \vdash t : A}{\Gamma \vdash \mu x:A.t : A}\ fix$$

$$\underbrace{\mu x:A.t}_{A} \equiv \underbrace{\underbrace{(\lambda x:A.t)}_{A\Rightarrow ?}\ \underbrace{(\mu x:A.t)}_{A}}_{?_A} : ?_1$$

$A \Rightarrow ?_1$

$$\overset{A}{\mu x:A.t} \rightarrow \overset{A}{t}[\underbrace{\mu x.A.t}_{A}/\underbrace{x}_{A}]$$

$$Fact = \mu f : nat\Rightarrow nat.\ \lambda n:nat.\ \_\_\_\_ \quad : nat \Rightarrow nat$$

$$\frac{\Gamma, x:A \vdash t:B}{\Gamma \vdash \lambda x:A.t : A\Rightarrow B}\Rightarrow_i$$

$$\frac{\Gamma\vdash t : A\Rightarrow B \qquad \Gamma\vdash r : A}{\Gamma \vdash t\,r : B}\Rightarrow_e$$

$$\vdash \lambda x:nat\Rightarrow nat.\ x\,((\lambda y:nat.\,y+2)\,3) : (nat\Rightarrow nat)\Rightarrow nat$$

$\Gamma = x:nat\Rightarrow nat,\ y:nat$

$$\dfrac{\dfrac{\dfrac{}{x:nat\Rightarrow nat \vdash x : nat\Rightarrow nat}\ ax_v \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash y:nat}\ ax_v \quad \dfrac{}{\Gamma\vdash 2:nat}\ ax_c}{x:nat\Rightarrow nat, y:nat \vdash y+2 : nat}\ +}{x:nat\Rightarrow nat \vdash \lambda y:nat.\,y+2 : nat\Rightarrow nat}\Rightarrow_i \quad \dfrac{}{x:nat\Rightarrow nat\vdash 3 : nat}\ ax_c}{x:nat\Rightarrow nat \vdash (\lambda y:nat.\,y+2)\,3 : nat}\Rightarrow_e}{x:nat\Rightarrow nat \vdash x\,((\lambda y:nat.\,y+2)\,3) : nat}\ \Rightarrow_e}{\vdash \lambda x:nat\Rightarrow nat.\ x\,((\lambda y:nat.\,y+2)\,3) : (nat\Rightarrow nat)\Rightarrow nat}\Rightarrow_i$$

$$((\lambda x.x)\,1)\ \lambda y.y$$

$$\dfrac{\dfrac{\dfrac{x:nat\vdash x : \cancel{(B\Rightarrow B)\Rightarrow A}\ nat}{\vdash \lambda x.x : nat \Rightarrow ((B\Rightarrow B)\Rightarrow A)} \quad \dfrac{}{\vdash 1 : nat}\ ax_c}{\vdash (\lambda x.x)\,1 : (B\Rightarrow B)\Rightarrow A} \qquad \dfrac{\dfrac{}{y:B\vdash y:B}\ ax_v}{\vdash \lambda y.y : B\Rightarrow B}\Rightarrow_i}{\vdash (\lambda x.x)\ 1\ \lambda y.y : A}\Rightarrow_e$$

No tree tho

$\lambda x : A . x\, 2$

$$\dfrac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A . t : A \Rightarrow B} \Rightarrow_i$$

$$\dfrac{\dfrac{\dfrac{}{x : nat \Rightarrow A \vdash x : nat \Rightarrow A}\, ax_v \qquad \dfrac{}{x : nat \Rightarrow A \vdash 2 : nat}\, ax_c}{x : nat \Rightarrow A \vdash x\, 2 : A} \Rightarrow_e}{\vdash \lambda x : nat \Rightarrow A . x\, 2 : (nat \Rightarrow A) \Rightarrow A} \Rightarrow_i$$

$$\dfrac{\Gamma \vdash t : A \Rightarrow B \qquad \Gamma \vdash r : A}{\Gamma \vdash t\, r : B}$$

$A = C$

$A = A \Rightarrow B$

$$\dfrac{\dfrac{\dfrac{}{x : A \vdash x : A \Rightarrow B}\, ax_v \qquad \dfrac{}{x : A \vdash x : A}\, ax_v}{x : A \vdash x\, x : B} \Rightarrow_e}{\vdash \lambda x : A . x\, x : A \Rightarrow B} \Rightarrow_i$$

Subject Reduction

$\boxed{\text{Preservación de tipos}}$

sujeto

$\left.\begin{array}{l}\Gamma \vdash t : A \\ t \to r\end{array}\right\} \Rightarrow \Gamma \vdash r : A$ SR

Predicado

$\Gamma \vdash A$

$\boxed{\text{PCF sin } \mu.}$ → SN

$\mu x : nat . x$

- Confluencia
- SR
- SN

SN → $\boxed{\Gamma \vdash t : A \text{ y } t \text{ no contiene } \mu \text{, } t \text{ es fuertemente normalizante}}$

$\Omega = (\lambda x . x x)(\lambda x . x x)$

Proof

\infer[A]{B}{C}

$$\dfrac{C}{B} A$$

$$\dfrac{C \qquad \dfrac{E}{D}}{B}$$

\infer{B}{
C & \infer[B]{E}
}

$$(\lambda x : nat \Rightarrow nat \,.\, x\ 1)(\lambda y : nat \,.\, y + 2)$$